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Error Propagation and Test Uncertainty
of LANL FTS Filter Test System
for Nuclear Storage Containers

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History of Revisions

Document Name	Effective Date	Action	Description
Moore ME 2015. Error Propagation and Test Uncertainty of LANL FTS Filter Test System for Nuclear Storage Containers R1.docx	8-26-2020	Minor revision	Revision to include newer experimental data, remove the word "canister" and replace it with "container", plus formatting changes.
Moore ME - Error Propagation - 2015 Test uncertainty calculation procedure for LANL FTS Filter Test System for nuclear storage canister filters.pdf	9-25-2015	New	Calculations for R&D determination of error propagation and test uncertainty of LANL FTS Filter Test System for nuclear storage containers.

This current document requires editing corrections on some equation expressions, for example in Section. 9.2.e Part 1.6.

Introduction. This document is a resource for ongoing work. This describes calculations for error propagation and test uncertainty of the LANL FTS Filter Test System for nuclear storage containers.

Note: In this document, any texts with quotations and in italics are from ASME-PTC-19.1-2005 "Test Uncertainty", and the section numbers in this document match the ASME-PTC-19.1-2005 format.

(9.2.a) Measurement Process

(9.2.a.1) "*Review test objectives and test duration*"

The Los Alamos FTS (Filter Test System) will measure performance quantities of the filters in nuclear storage containers. Two performance quantities are measured,

(9.2.a.1.1) A container filter must have a measured aerosol capture percent of at least 99.97% when challenged with a 0.45 µm mean diameter PAO (polyalphaolefin) oil aerosol (Moore 2014). This corresponds to a leak percent of 0.03% ($3.0*10^{-2}$ %).

(9.2.a.1.2) The test air flow shall be 200 accm (volumetric cubic centimeters per minute of air flow). At this air flow rate, the measured pressure drop across a filter shall not exceed 0.25 kPa (1.0 inWC). An individual test of a filter requires about 15 minutes of duration in the LANL FTS device. These tests will be performed over the course of at least a five year period of time.

(9.2.a.2) "*List all independent measurement parameters and their nominal levels for the test*"

Principle Instruments Calibration Verification FTS-SNMC TA-03 List.xlsx				
Quantity	Principle instrument list (mfg) .	Mfg. & model No.	Serial No.	Calibration / verification
C(µg/L)	Aerosol photometer (ATI Inc)	#2HN	21772	Mfg calibration (ATI Inc).
Q(accm)	Flow controller (Omega Inc)	FMA-2605A-V2	89557	Bios Defender 530M SN 123014
ΔP(inWC)	Pressure gauge (Furness Inc)	FCO332	1211052	Dwyer 400-5-L LANL S&CL 101393

For a filter test, the mass concentration of the (challenge) upstream oil droplet aerosol must have a magnitude of 65 +/- 15 micrograms per liter of air,

The leak percent, P%, is the percent ratio of downstream (C_D) to upstream (C_U) aerosol concentration, where,

$$P\% = 100\% * C_D / C_U$$

The model 2HN photometer (ATI Test Inc., Owings Mill, MD) measures aerosol concentration, and provides a digital panel display output of concentration in terms of micrograms of aerosol per liter of air C(µg/L). The 2HN photometer has a dynamic range from "0.00005 to 120 µg/L".

Independent parameter	Symbol (units)	Nominal magnitude	Comment
C_U = upstream aerosol concentration ratio	C_U(µg/L)	C_U must be between 65 ±15 µg/L for testing.	This concentration is a necessary condition.
Flow rate (this can be a correlated uncertainty)	Q(accm)	The air flow must be controlled to 200 +/- 1 cc per minute.	The flowrate is a quantity that correlates the leak measure and the pressure drop.

(9.2.a.3) "*Calibrations and instrument setups that will affect each parameter*"

Uncertainties in measurement system components can affect two or more measurements simultaneously (correlated uncertainties).

Calibrations that will affect each parameter

Three devices that must be calibrated are included in each FTS device:

(A) a photometer that measures the mass concentration of oil droplet aerosol in the air to a magnitude of 65 +/- 15 micrograms of oil aerosol per liter of air,

(B) an electronic pressure gauge with nominal resolution +/-0.01 inches H2O WC between 0.0 and 1.0 inches H2O WC, and,

(C) a flow controller-meter which maintains air flow at 200 +/- 1 cc per minute (nominal resolution).

Instrument setups that will affect each parameter

(A) A leak in the FTS system setup can introduce errors in measuring: filter leak percent, filter pressure drop and in air flow rate determination.

(B) Successive filter tests with oil droplet aerosol can clog a filter that is being tested, and the measured leak percent and pressure drop are affected. (Moore 2012)

	Quantity	Symbol	Units	Type of variable
v_1	Flow	Q	mL/min	Independent variable
v_2	Pressure	ΔP	inWC	Dependent variable
v_3	Concentration (downstream)	C_D	µg/L	Dependent variable
v_4	Concentration (upstream)	C_U	µg/L	Dependent variable

Dependent parameter	Symbol (units)	Nominal magnitude	Comment
C_D = downstream aerosol concentration ratio	C_D(µg/L)	C_D must be measured on a range from 0.65 to 0.0065 µg/L.	For C_U = 65 µg/L, these C_D values correspond to PEN% of 1.0% and 0.01%.
Pressure drop	ΔP(inWC) or ΔP(kPa)	0.5 to 1.0 in WC, or 0.125 to 0.25 kPa	This is a correlated quantity with leak percent (via air flow rate.)

(9.2.a.4) "*The functional relationship between the independent measurement parameters and the test result*"

For the FTS, there are two test results: the leak percent, P%, and the filter pressure drop, ΔP. There is also dependent air flow rate parameter, Q(accm).

<u>Parameter #1 (P%, leak percent)</u>
For P% (leak percent),
$P\% = 100\% * C_D / C_U$
C_D = downstream aerosol concentration
C_U = upstream aerosol concentration

The aerosol concentration measurement is performed by the ATI model 2HN photometer, and the photometer flowrate is set by the Omega flow controller. It is assumed that any individual measurement of the aerosol leak percent would experience an identical fluctuation of air flow for both the upstream and downstream aerosol concentration measurements. Since the leak percent is a ratio of the two measured concentrations, it is assumed that any fluctuation would be arithmetically removed. Therefore, the influence of flowrate fluctuations are ignored for measurements of the leak percent. However, the air flow fluctuations will not be ignored for the measurement of the filter pressure drop.

C_D = downstream aerosol concentration ratio)	P% = 100% * C_D(µg/L) / C_U(µg/L)	C_D or C_U range from 0.00005 to 120 µg/L.	This is a correlated quantity with pressure drop (via air flow rate.)

However, these three quantities that were selected are not independent variables for the function (penetration).

A photometer measures a concentration in a flowing volume of air, and penetration can be expressed:

$P\% = LEAK\% = 100\% * C_D(\mu g/L) / C_U(\mu g/L)$

Parameter #2 (Filter pressure drop, ΔP)
Filter pressure drop, ΔP

For the range of operation of the FTS Filter Test System, assume the relation between pressure drop and flow rate is linear.

ΔP(inWC) = k*Q(accm), where

k = a constant unique to the FTS system, with units of (inWC/accm).

Parameter #3 (Air flow, Q)

For the air flow, the following has been quoted:

> **Ref:** *(Moore ME, Reeves, KP, 2013. Filter Measurement System for Nuclear Material Storage Canisters - End of Year Report FY 2013. Los Alamos National Laboratory, Los Alamos Unclassified Report LAUR-14-20641.)*
>
> ***Appendix C.***
> ***Air Flow (Controller) Setpoint***
> ***Example:*** *To set the actual test flowrate, Q_{TA}, through the tested filter at 0.200 ALPM, the setpoint air flowrate of the flow controller must be determined with respect to the ambient air pressure in the local environment of the test system.*
> *The listing of the definitions of the quantities are given below, where,*
>
> > *Q_{TA} = the test flowrate in terms of "actual" units, ALPM, actual liters per minute (e.g. 0.200 ALPM),*
> > *Q_{FS} = the air flowrate in the Omega Inc. flow controller FIC102, in "standard" units, SLPM,*
> > *P_A = the "actual" air pressure in the room air of the test system (e.g. 11.2 psia in Los Alamos), it must be emphasized this is not the air pressure inside the flow controller, and,*
> > *P_S = the "standard" air pressure at sea level (i.e. 14.7 psia), then by extension,*
> >
> > *$Q_{FS} = Q_{TA} * (P_A / P_S)$, or*
> > *Q_{FS} = 0.152 SLPM = 0.200 ALPM * (11.2 psia / 14.7 psia),*
>
> ***Therefore, the setpoint of the flow controller needs to be 0.152 SLPM, assuming a desired actual air flowrate of 0.200 ALPM, and a local room air pressure of 11.2 psia. Recent practice has been setting the flow controller setting at "0.149 SLPM air flow".***

In this error uncertainty calculation, define Q_{CS} = the corrected air flow in the Omega flow controller in units of SLPM, where

$Q_{CS} = a_0 + a_1 * Q_{FS}$
Q_{FS} = the indicated Omega flow controller (SLPM)

$a_1 = Q_{CS} / Q_{FS}$ = 0.152 SLPM / 0.149 SLPM = 1.020, and
a_1 = 0.0

(9.2.b) "**List Elemental Error Sources (see subsection 5-3)"**

> ***"5-3 Identification Of Error Sources"***
> *"Once the true value has been defined, the errors associated with measuring the true value must be identified. Examples of error sources include imperfect calibration corrections, uncontrolled test conditions, measurement methods, environmental conditions, and data reduction techniques."*

(9.2.b.1) Make a complete and exhaustive list of all possible test uncertainty sources for all parameters.

(A.) v_1 = Q(accm) = air flow. Calibration corrections, system air leakage (uncontrolled test condition), and temperature corrections (environmental conditions).

(B.) v_2 = ΔP(inWC) = Pressure drop. Calibration corrections, system air leakage (uncontrolled test condition), and temperature corrections (environmental conditions).

(C.) v_3 = C_U(μg/L) = Concentration of aerosol (upstream). Calibration corrections, and system air leakage (uncontrolled test condition).

(D.) v_4 = C_D(μg/L) = Concentration of aerosol (upstream). Calibration corrections, and system air leakage (uncontrolled test condition).

(9.2.c) ***"Calculate the Systematic Uncertainty and Random Uncertainty (Standard Deviation of the Mean) for Each Parameter (see subsections 6-1 and 6-2)."***

Random Uncertainty
Review the definition of the *"Random Uncertainty (Standard Deviation of the Mean)"*

Using a pressure transducer as an example, the random uncertainty is composed from a population of individual measurements from the instrument.

When N_P previous values (X_{Pj}) are known for the quantity being measured, the sample standard deviation for the variable can be calculated as

$$s_X = s_{X_P} = \left[\frac{1}{N_P - 1}\sum_{j=1}^{N_P}(X_{P_j} - \overline{X}_P)^2\right]^{1/2} \qquad (6\text{-}1.2)$$

where

$$\overline{X}_P = \frac{1}{N_P}\sum_{j=1}^{N_P} X_{P_j} \qquad (6\text{-}1.3)$$

The appropriate random standard uncertainty of the mean for the current measurement ($\overline{X}$) is then

$$s_{\overline{X}} = \frac{s_X}{\sqrt{N}} \qquad (6\text{-}1.4)$$

where N is the number of current measurements averaged to determine $\overline{X}$. The number of degrees of freedom for this random standard uncertainty of the mean $s_{\overline{X}}$ is

$$\nu = N_P - 1 \qquad (6\text{-}1.5)$$

This estimate of the random standard uncertainty is an ISO Type A estimate since it is obtained from data. The case where the data sample is only a single measurement is handled above with $N = 1$.

Note: Excel STDEV = S_X ; identical to Eq. 6-1.2 from ASME PTC 19.1-2005 “Test Uncertainty”

$\boxed{\sqrt{\frac{\sum (x-\overline{x})^2}{(n-1)}}}$, and,

The Excel AVERAGE function is identical to Eq. 6-1.3 from ASME PTC 19.1-2005 “Test Uncertainty”

Systematic Uncertainty

Using a pressure transducer as an example, the systematic uncertainty could be the manufacturer’s published uncertainty.

6-2 SYSTEMATIC STANDARD UNCERTAINTY OF A MEASUREMENT

The systematic standard uncertainty $b_{\overline{X}}$ of a measurement was defined in para. 4-3.2 as a value that quantifies the dispersion of the systematic error associated with the mean. The true systematic error (β) is unknown, but $b_{\overline{X}}$ is evaluated so that it represents an estimate of the standard deviation of the distribution for the possible β values. It should be noted that while $b_{\overline{X}}$ is an estimate of the dispersion of the systematic errors in a measurement, the systematic error that is present in a specific measurement is a fixed single value of β.

The systematic standard uncertainty of the measurement is the root-sum-square of the elemental systematic standard uncertainties $b_{\overline{X}_k}$ for all sources.

$$b_{\overline{X}} = \left[\sum_{k=1}^{K} (b_{\overline{X}_k})^2\right]^{1/2} \qquad (6\text{-}2.1)$$

where

K = the total number of systematic error sources

and each

$b_{\overline{X}_k}$ = an estimate of the standard deviation of the k^{th} elemental error source

6-4 COMBINED STANDARD AND EXPANDED UNCERTAINTY OF A MEASUREMENT

For simplicity of presentation, a single value is often preferred to express the estimate of the error between the mean value ($\overline{X}$) and the true value, with a defined level of confidence. The interval

$$\overline{X} \pm U_{\overline{X}} \qquad (6\text{-}4.1)$$

represents a band about $\overline{X}$ within which the true value is expected to lie with a given level of confidence (see Fig. 4-3.3). The uncertainty interval is composed of both the systematic and random uncertainty components.

The general form of the expression for determining the uncertainty of a measurement is the root-sum-square of the systematic and random standard uncertainties of the measurement, with this quantity defined as the combined standard uncertainty ($u_{\overline{X}}$) [1].

$$u_{\overline{X}} = \sqrt{(b_{\overline{X}})^2 + (s_{\overline{X}})^2} \qquad (6\text{-}4.2)$$

where

$b_{\overline{X}}$ = the systematic standard uncertainty [eq. (6-2.1)]

$s_{\overline{X}}$ = the random standard uncertainty of the mean [eq. (4-3.3), (6-1.4), or (6-1.6) as appropriate]

In order to express the uncertainty at a specified confidence level, the combined standard uncer-

19

(9.2.d*) "Propagate the Systematic and Random Standard Deviations (see subsections 7-1 through 7-4)"*

(1) The systematic uncertainty and random uncertainty (sample standard deviations of the means) of the independent parameters are propagated separately all the way to the final result.

(2) Propagation of the standard deviations of the means is done, according to the functional relationship defined in step (a)(4), by using the Taylor series method (see section 7). This requires a calculation of the sensitivity factors, either by differentiation or by numerical analysis.

(9.2.e) ***"Calculate Uncertainty (see subsection 7-5)"***

(1) Combine the systematic and random uncertainties to obtain the total uncertainty.

6-4 COMBINED STANDARD AND EXPANDED UNCERTAINTY OF A MEASUREMENT

For simplicity of presentation, a single value is often preferred to express the estimate of the error between the mean value ($\overline{X}$) and the true value, with a defined level of confidence. The interval

$$\overline{X} \pm U_{\overline{X}} \qquad (6\text{-}4.1)$$

represents a band about $\overline{X}$ within which the true value is expected to lie with a given level of confidence (see Fig. 4-3.3). The uncertainty interval is composed of both the systematic and random uncertainty components.

The general form of the expression for determining the uncertainty of a measurement is the root-sum-square of the systematic and random standard uncertainties of the measurement, with this quantity defined as the combined standard uncertainty ($u_{\overline{X}}$) [1].

$$u_{\overline{X}} = \sqrt{(b_{\overline{X}})^2 + (s_{\overline{X}})^2} \qquad (6\text{-}4.2)$$

where

$b_{\overline{X}}$ = the systematic standard uncertainty [eq. (6-2.1)]

$s_{\overline{X}}$ = the random standard uncertainty of the mean [eq. (4-3.3), (6-1.4), or (6-1.6) as appropriate]

In order to express the uncertainty at a specified confidence level, the combined standard uncer-

(9.2.e) "*Calculate Uncertainty (see subsection 7-5)*"

(9.2.e.1) Air flow – flow controller – (independent parameter)

Calculate the combined standard uncertainty, $U_{\bar{Y}}$,

$U_{\bar{Y}} = [(b_{\bar{Y}})^2 + (S_{\bar{Y}})^2]^{½}$ where,

$(b_{\bar{Y}})$ = all forms of systematic uncertainty

$(S_{\bar{Y}})$ = random standard uncertainty of the mean.

(1.1) Define the calibration range of the air flow controller, e.g. from 180 to 220 accm.

(1.2) Calibrate the flow controller for at least five flowrates, 0.134, 0.142, 0.149, 0.156 and 0.164 SLPM.

These flowrates correspond to 0.180, 0.190, 0.200, 0.210 and 0.220 ALPM, respectively.

(1.3) Organize the primary and secondary flowmeter data in the same format as the figure below

(Ref: *FTS Flow SLPM total uncertainty.xlsx*)

(1.4) The $(s_{\bar{Y}})$ = random standard uncertainty of the mean is associated with the y-axis dependent variable.

(1.5) $(S_{\bar{Y}})$ = measured random standard uncertainty of the mean = __±________________(accm).

(1.6) There are two contributions to the b□ , systematic uncertainty:

(1.5.1) b□1 (manufacturer listed uncertainty) = __±__0.8% of a reading (of_0.149 SLPM) or 0.0012 SLPM, and

(1.5.2) b□2 (observed fluctuation of the flow controller digital display)= _±0.001 SLPM _.

For the flow controller, the combined standard uncertainty is,

$U_{\bar{Y}} = [(b_{\bar{Y}1})^2+(b_{\bar{Y}2})^2+ (S_{\bar{Y}})^2]^{½}$ = __±________________(SLPM)_.

FTS Flow SLPM total uncertainty.xlsx

Secondary SLPM Omega controller	Primary SLPM Bios calibrator	Equi-distant x-values	$S_{\hat{Y}}$	Lower bound of uncertainty	Upper bound of uncertainty	
0.134	0.1414	0.134	0.0006	0.1405	0.1417	5.37
0.142	0.1497	0.142	0.0004	0.1485	0.1493	5.28
0.149	0.1556	0.149	0.0003	0.1564	0.1571	4.33
0.156	0.1644	0.157	0.0004	0.1642	0.1651	5.24
0.164	0.1728	0.164	0.0006	0.1719	0.1731	5.23

Step 1. Calculate the "SEE" = STEYX.

$$SEE = \left[\frac{\sum_{j=1}^{N}(Y_j - mX_j - c)^2}{N-2}\right]^{1/2} \qquad (8\text{-}6.4)$$

***The primary measuring instrument is the DEPENDENT (y-axis) variable.

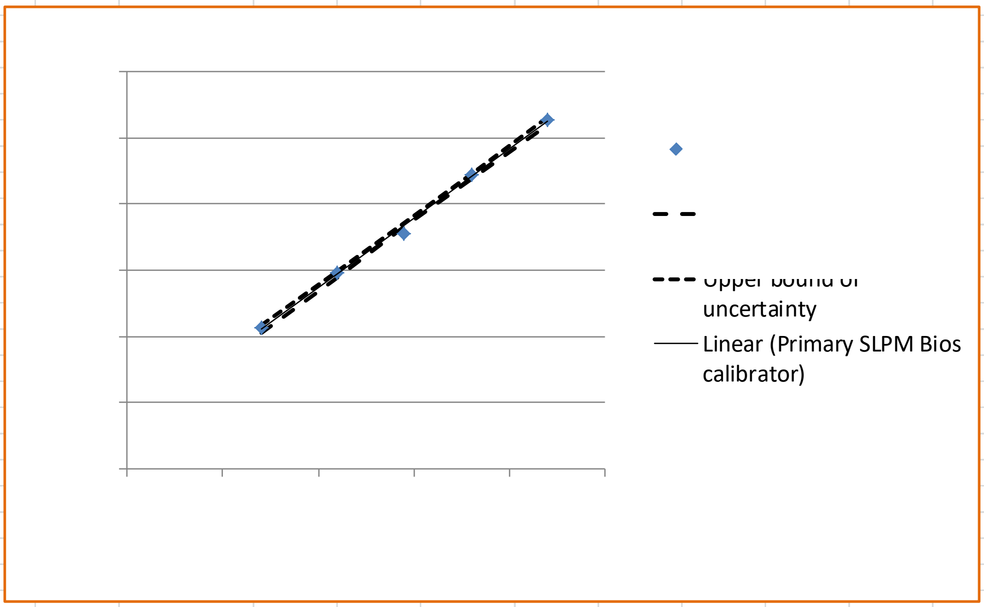
For a given value of X, the random standard uncertainty associated with the $\hat{Y}$ obtained from the curve-fit [eq. (8-6.1)] is

$$s_{\hat{Y}} = SEE\left[\frac{1}{N} + \frac{(X-\bar{X})^2}{\sum_{j=1}^{N}(X_j-\bar{X})^2}\right]^{1/2} \qquad (8\text{-}6.5)$$

0.00076	= SEE = STEYX(B3:B7,A3:A7)
5	= N = number of samples
0.149	= $\bar{X}$ = average of x-values
0.01047	= σ_P = STDEV.P(A3:A7) = $[\Sigma(X_j - \bar{X})^2 / n]^{1/2}$
0.149	= Input here a given value of "X".
0.00034	= $S_{\hat{Y}}$ = SEE*$[1/N + (X-\bar{X})^2/(N*\sigma_P*\sigma_P)]^{1/2}$; the random standard uncertainty associated with the $\hat{Y}$ obtained from the curve-fit.
1.04726	= SLOPE function from Excel
0.00074	= INTERCEPT function from Excel

Primary SLPM Bios calibrator ____ model and SN

Secondary SLPM Omega controller ____ model and SN



Ref. Milton and Arnold 1986 ex.11.3.3.xlsx

cf. ASME-PTC-19.1-2005 Test Uncertainty

Combined standard uncertainty (for C_U)

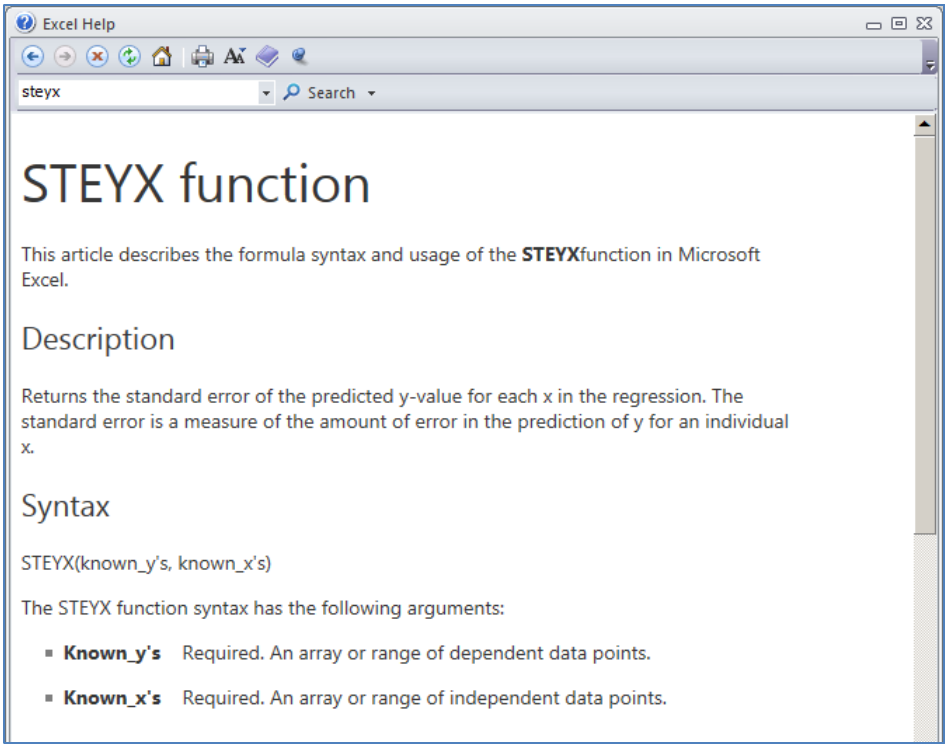
$U_{\bar{X}} = [(b_{\bar{X}})^2 + (S_{\bar{X}})^2]^{½}$ where,

$b_{\bar{X}} = [(b_{\bar{X}1})^2 + (b_{\bar{X}2})^2 + (b_{\bar{X}3})^2 + ...]^{½}$ where,

$S_{\bar{X}} = S_X / (N)^{1/2}$ (ASME PTC 19.1 Eq. 6-1.4)

	(SLPM)	(SLPM)	(SLPM)	(SLPM)	(SLPM)	
	$(b_{\bar{Y}1})$	$(b_{\bar{Y}2})$	$(b_{\bar{Y}})$ total	$(b_{\bar{Y}})$ total	$S_{\bar{Y}}$	$U_{\bar{Y}}$ (SLPM)
SLPM	0.0012	0.0010	0.00156	0.00156	**0.00034**	**0.0016**

$(b_{\bar{Y}1})$	*notes - Error propagation 2015 FTS LANL.docx*
$(b_{\bar{Y}2})$	*notes - Error propagation 2015 FTS LANL.docx*
$S_{\bar{Y}}$	*Omega flow controller - rand std uncert SY - Feb 2015.xlsx*



Excel Help

steyx

Search

STEYX function

This article describes the formula syntax and usage of the **STEYX**function in Microsoft Excel.

Description

Returns the standard error of the predicted y-value for each x in the regression. The standard error is a measure of the amount of error in the prediction of y for an individual x.

Syntax

STEYX(known_y's, known_x's)

The STEYX function syntax has the following arguments:

- **Known_y's** Required. An array or range of dependent data points.
- **Known_x's** Required. An array or range of independent data points.

(9.2.e.2) Filter pressure drop - Pressure transducer – (dependent parameter).

Perform a calibration of the FTS Filter Test System pressure transducer (secondary instrument) against a primary standard (e.g. inclined manometer).

(2.1) Define the calibration range of the air flow controller, e.g. from 0.4 to 1.2 inWC.

(2.2) Calibrate the flow controller for at least five flowrates, e.g. 0.4, 0.6, 0.8, 1.0 and 1.2 inWC.

Organize the primary and secondary pressure data in the same format (c.f. *Uncertainty Honeywell 15psi - random standard SY - use ASME-PTC-19.1-2005.xlsx*)

(2.3) ($S_{\bar{Y}}$) = measured random standard uncertainty of the mean = __±_0.0274________(inWC).

(2.4) There are two contributions to the b□ , systematic uncertainty:

(1.5.3) b□1 (manufacturer listed uncertainty) = _±_0.0024 inWC   (0.3%), and

(1.5.4) b□2 (observed uncertainty for reading the inclined manometer)= _±_0.005 inWC _.

For the pressure transducer, the combined standard uncertainty is,

$U_{\bar{Y}} = [(b_{\bar{Y}1})^2+(b_{\bar{Y}2})^2+ (S_{\bar{Y}})^2]^{½}$ = __±_0.0279__________(inWC)_.

Error propagation 2015 FTS LANL 1.2.xlsx — Data from: *FTS error uncertainty data - Feb 2015.xlsx*

Secondary Pressure Gauge - Furness - inWC	Primary Gauge - Dwyer - inWC - avg (ΔH)	Equi-distant x-values	$S_{\hat{Y}}$	Lower bound of uncertainty	Upper bound of uncertainty	
0.39	0.3744	0.39	0.0236	0.3554	0.4026	4.08
0.61	0.5991	0.595	0.0169	0.5641	0.5979	1.80
0.81	0.8087	0.800	0.0135	0.7695	0.7966	0.16
1.1	1.0371	1.005	0.0157	0.9693	1.0008	5.89
1.21	1.214	1.21	0.0219	1.1652	1.2089	0.33

$$SEE = \left[\frac{\sum_{j=1}^{N}(Y_j - mX_j - c)^2}{N-2}\right]^{1/2} \qquad (8\text{-}6.4)$$

Step 1. input data and calculate the "SEE" = STEYX.

For a given value of X, the random standard uncertainty associated with the $\hat{Y}$ obtained from the curve-fit [eq. (8-6.1)] is

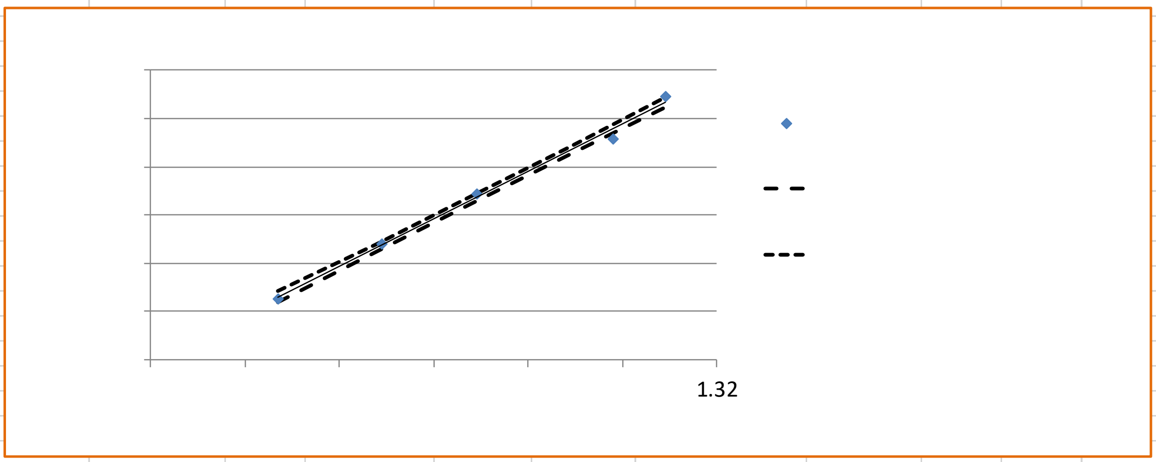
$$s_{\hat{Y}} = SEE\left[\frac{1}{N} + \frac{(X-\overline{X})^2}{\sum_{j=1}^{N}(X_j-\overline{X})^2}\right]^{1/2} \qquad (8\text{-}6.5)$$

0.03020	= SEE = STEYX(B3:B7,A3:A7)
5	= N = number of samples
0.824	= $\overline{X}$ = average of x-values
0.30316	= σ_P = STDEV.P(A3:A7) = $[\Sigma(Xj - \overline{X})^2 / n]^{1/2}$
0.8	= Input here a given value of "X".
0.01355	= $S_{\hat{Y}}$ = SEE*$[1/N + (X-\overline{X})^2/(N*\sigma_P*\sigma_P)]^{1/2}$; the random standard uncertainty with $\hat{Y}$ obtained from the curve-fit.
0.98543	= SLOPE function from Excel
-0.00533	= INTERCEPT function from Excel

User1: changed - see pg. 62 - logbook NucFilt #5.

Primary pressure gauge calibrator ____________________________ model and SN

Secondary Furness pressure gauge ____________________________ model and SN



Ref. Milton and Arnold 1986 ex.11.3.3.xlsx — cf. ASME-PTC-19.1-2005 Test Uncertainty

Combined standard uncertainty (for C_U)

$U_{\overline{X}} = [(b_{\overline{X}})^2 + (S_{\overline{X}})^2]^{1/2}$ where,

$b_{\overline{X}} = [(b_{\overline{X}1})^2 + (b_{\overline{X}2})^2 + (b_{\overline{X}3})^2 + \ldots]^{1/2}$ where,

$S_{\overline{X}} = S_X / (N)^{1/2}$ (ASME PTC 19.1 Eq. 6-1.4)

	(inWC)	(inWC)	(inWC)	(inWC)	(inWC)		
	$(b_{\bar{Y}1})$	$(b_{\bar{Y}2})$	$(b_{\bar{Y}})$ total	$(b_{\bar{Y}})$ total	$S_{\hat{Y}}$	$U_{\bar{Y}}$ (inWC)	$U_{\bar{Y}}$ (inWC)
SLPM	0.0024	5.00E-03	5.55E-03	5.55E-03	0.01355	0.0146	0.0146

$(b_{\bar{Y}1})$	*Uncertainty - Furness FCO332 measurement*
$(b_{\bar{Y}2})$	*Uncertainty - Furness FCO332 voltage*
$S_{\hat{Y}}$	*Error propagation 2015 FTS LANL 1.2.xlsx*

(3) C_U = upstream aerosol concentration (independent parameter)

Aerosol photometer (P% or Pen% aka L% or Leak%) measurement.

For the independent parameter, C_U, calculate the combined standard uncertainty, $U_{□}(C_U)$

$U_{□} = [(b_{□})^2 + (S_{□})^2]^{½}$ where,

$(b_{□})$ = all forms of systematic uncertainty

$b_{□} = [(b_{□1})^2 + (b_{□2})^2 + (b_{□3})^2 + \ldots]^{½}$ where,

$(S_{□})$ = random standard uncertainty of the mean.

<u>From:</u> Model 2HN photometer (Air Techniques International) Manual pg. 30 P/N 1800110, Rev. H

<u>Dynamic Range:</u> 0.00005 to 120 micrograms per liter (µg/L).

<u>Accuracy:</u> 1% full-scale for the amplifier decade in use.

<u>Repeatability:</u> 0.5% full-scale for the amplifier decade range in use.

* Systematic Uncertainty

$b_{□1}(C_U(\text{accuracy}))$ = 65 µg/L *0.01 = 0.65 µg/L

$b_{□2}(C_U(\text{repeatability}))$ = 65*0.005 = 0.325 µg/L

* Random Standard Uncertainty

First take an initial sample of the C_U value, with a number, N_P, of readings. This is not the measurement of the C_U value in regular testing. This value of SX is taken at a separate (previous) time, i.e. it does not have to be performed at the same time as the regular (current) measurements.

Calculate the sample standard deviation, S_X, for these N_P number of samples.

$$S_X = [\,(1/(N_P-1)\,\Sigma(X_{PJ} - □_P)^2]^{1/2} \qquad \text{(ASME PTC 19.1 Eq. 6-1.2)}$$

For Excel, this is the same as the STDEV function.

However, when regular testing with the FTS occurs, then a different number of samples, N, are taken to measure the average value of C_U.

The appropriate random standard uncertainty of the mean for the regular (current) measurement is then:

$$S_{□} = S_X / (N)^{1/2} \qquad \text{(ASME PTC 19.1 Eq. 6-1.4)}$$

Use the Excel spreadsheet *FTS error uncertainty data - Feb 2015.xlsx*
to calculate $S_{□}$.

$S_{□}$. = 1.21 (this was the STDEV based on 100 measurements that had an average of 60.35 µg/L.

(4) C_D = downstream aerosol concentration (dependent parameter)

From: Model 2HN photometer (Air Techniques International) Manual pg. 30 P/N 1800110, Rev. H
Dynamic Range: 0.00005 to 120 micrograms per liter (μg/L).
Accuracy: 1% full-scale for the amplifier decade in use.
Repeatability: 0.5% full-scale for the amplifier decade range in use.

* Systematic Uncertainty
$b_{\bar{Y}1}(C_D(\text{accuracy})) = 0.0002$ μg/L $*0.01 = 2*10^{-6}$ μg/L
$b_{\bar{Y}2}(C_D(\text{repeatability})) = 0.0002*0.005 = 1*10^{-6}$ μg/L

Choose $C_D = 0.0002$ μg/L (two decades smaller than the criterion),
because for $P\% \leq 0.03\%$, $C_D = 0.0003*65$ μg/L = 0.0195 μg/L.

Use the Excel spreadsheet *FTS error uncertainty data - Feb 2015.xlsx*
to calculate $S_{\bar{Y}}$.
$S_{\bar{Y}}$. = 0.00076 (this was the STDEV based on 100 measurements that had an average of 0.00055 μg/L.

Summary of Data – Calculated Result

(1) Reported Aerosol Percent Leak (percent penetration)
From ASME-PTC-19.1-2005 (Table C-4), there is a defined form of the uncertainty that can be applied to the measurement of PEN%.

For P% = PEN%,
$P\% = 100\% * C_D / C_U$
C_D = downstream aerosol concentration
C_U = upstream aerosol concentration
$U_R = [(100\%\ U(C_D) / C_U)^2 + (100\%\ C_D * U(C_U) / C_U^2)^2]^{1/2}$
For the dependent parameter, C_D, calculate the combined standard uncertainty, $U_{\bar{Y}}(C_D)$
$U_{\bar{Y}} = [(b_{\bar{Y}})^2 + (S_{\bar{Y}})^2]^{1/2}$ where,
$(b_{\bar{Y}})$ = all forms of systematic uncertainty
$(S_{\bar{Y}})$ = random standard uncertainty of the mean.

FTS TOTAL PEN test uncertainty.xlsx

FTS CU total uncertainty.xlsx

Combined standard uncertainty (for C_U)

$U_{\bar{X}} = [(b_{\bar{X}})^2 + (S_{\bar{X}})^2]^{½}$ where,

$b_{\bar{X}} = [(b_{\bar{X}1})^2 + (b_{\bar{X}2})^2 + (b_{\bar{X}3})^2 + \ldots]^{½}$ where,

$S_{\bar{X}} = S_X / (N)^{1/2}$ (ASME PTC 19.1 Eq. 6-1.4)

	$(b_{\bar{X}1})$	$(b_{\bar{X}2})$	$(b_{\bar{X}})$ total	$(b_{\bar{X}})$ total	$S_{\bar{X}}$	$S_{\bar{X}}$		$U_{\bar{X}}\ (C_U)$
µg/L	0.65	0.325	0.727	0.727	1.207	1.207		**1.41**

FTS CD total uncertainty.xlsx

Combined standard uncertainty (for C_D)

$U_{\bar{Y}} = [(b_{\bar{Y}})^2 + (S_{\bar{Y}})^2]^{½}$ where,

$b_{\bar{Y}} = [(b_{\bar{Y}1})^2 + (b_{\bar{Y}2})^2 + (b_{\bar{Y}3})^2 + \ldots]^{½}$ where,

$S_{\bar{Y}} = S_Y / (N)^{1/2}$ (ASME PTC 19.1 Eq. 6-1.4)

	$(b_{\bar{Y}1})$	$(b_{\bar{Y}2})$	$(b_{\bar{Y}})$ total	$(b_{\bar{Y}})$ total	$S_{\bar{Y}}$	$S_{\bar{Y}}$	$U_{\bar{Y}}\ (C_D)$	$U_{\bar{Y}}\ (C_D)$
µg/L	2.0E-06	1.0E-06	2.24E-06	2.24E-06	7.64E-04	7.64E-04	7.64E-04	**7.64E-04**

Combined standard uncertainty (for reported P%)

$U_R = [(100\%\ U(C_D) / C_U)^2 + (100\%\ C_D * U(C_U) / C_U^2)^2]^{½}$

	$x = C_D$	$y = C_U$	$U_x = U(C_D)$	$U_y = U(C_U)$		PEN%	$\pm u_R$
µg/L	0.00055	60.35	7.64E-04	1.41		9.11E-04	1.27E-03

(2) Reported Filter Pressure Drop (inWC)

Error propagation 2015 FTS LANL 1.2.xlsx							
	u(reported pressure drop) = u(pressure gauge) + m*u(flow meter)						
	u (pressure gauge, inWC)	m = slope of flow meter vs pressure gauge	u (flow meter, SLPM)	u (reported pressure drop * slope, inWC)		u (reported pressure drop, inWC) Absolute Combined Standard Uncertainty	u (reported) Absolute Expanded Uncertainty, $U_R = 2*u_R$
	0.0279	5.1007	1.60E-03	8.16E-03		2.91E-02	5.81E-02
	Absolute Systematic Standard Uncertainty b_R			**Absolute Random Standard Uncertainty s_R**		**Absolute Combined Standard Uncertainty u_R**	**Absolute Expanded Uncertainty $U_R = 2*u_R$**

Error propagation 2015 FTS LANL 1.2.xlsx							
			Table of Data				
			Independent Parameters				
Symbol	**Description (Manufacturer and Instrument serial number)**	**Units**	**Nominal Value**	**Absolute Systematic Standard Uncertainty b_{Xi}**	**Absolute Random Standard Uncertainty S_{Xi}**	**Absolute Combined Standard Uncertainty u_R**	
C_D	Aerosol concentration (ATI 2HN SN-21772)	(µg/L)	0.0195	2.24E-06	7.64E-04		
C_U	Aerosol concentration (ATI 2HN SN-21772)	(µg/L)	65.0	0.73	1.21		
Q	Air flow Omega FMA-2605A-V2 SN-89557	(SLPM)	0.152	1.56E-03	3.41E-04		
ΔP (instru - ment)	Pressure drop Furness FCO332 SN-1211052	(inWC)	0.70	5.55E-03	1.35E-02	1.46E-02	
			Summary of Data				
			Calculated Result				
Symbol	**Description**	**Units**	**Calculated Result, R**	**Absolute Systematic Standard Uncertainty b_R**	**Absolute Random Standard Uncertainty s_R**	**Absolute Combined Standard Uncertainty u_R**	**Absolute Expanded Uncertaint y $U_R = 2*u_R$**
P%	Aerosol leak (penetration)	Percent (ratio)	9.11E-04	n/a	n/a	1.27E-03	2.53E-03
ΔP (filter report)	Filter pressure drop - reported by FTS system	(inWC)	0.70	0.0146	8.16E-03	2.91E-02	5.81E-02

References

ASME American Society of Mechanical Engineers. 2005. Test Uncertainty. ASME–PTC 19.1-2005.
Fox RW and AT McDonald 1985. Introduction to Fluid Mechanics. Appendix. Analysis of Experimental Uncertainty

Moore ME. 2014. Evaluating the use of PAO (4 cSt polyalphaolefin) oil instead of DOP (di-octyl phthalate) oil for measuring the aerosol capture of nuclear canister filters. LA-UR- 14-25489. Los Alamos National Laboratory.

Moore ME. 2012. Lifetime Extension for SNMC (SAVY) Canister Filters DOE Manual 441.1-1 Working Group Meeting. Albuquerque, NM June 12, 2012. LAUR-12-22078.

Spreadsheet resources

Moore ME. 2013. Excel spreadsheet. Pressure drop measurement - reconciled - LANL vs. NucFilt filter - flow controller 8-15-13.docx. Los Alamos National Laboratory

Moore ME 2014 calibration propagation of errors reference.xlsx

pg. 141 LANL NucFilt logbook #1, pp. 141-145; pp. 152-153	20-Jun-11
LANL_Propagation of Errors_for NFT Filters-rev2.xlsx	22-Jun-11
pg. 135-6 LANL NucFilt logbook #3, pp. 135-136	16-Jan-13
NQA-1 calibration pressure spreadsheet.xlsx	29-Jan-13

__**Appendix**__

$$r = kx^a y^b \qquad S_r^2 = (aky^b x^{a-1} S_x)^2 + (bkx^a y^{b-1} S_y)^2$$